The 6th International Federation of Nonlinear Analysts Conference (2012) 48–56

Singularly Impulsive Dynamical Systems with Time Delay: Lyapunov-Krasovskii Stability

Nataša A. Kablar [1], Vladimir Kvrgić[2], Mirko Bućan [3], and Dragutin Lj. Debeljković [4]
[1]N. A. Kablar is with Lola Institute and Faculty of Computer Science, 11000 Belgrade, Serbia.
[2]V. Kvrgić is with Lola Institute, 11000 Belgrade, Serbia.
[3]M. Bućan is with Lola Institute, 11000 Belgrade, Serbia.
[4]D. Lj. Debeljković is with Faculty of Mechanical Engineering, University of Belgrade, 11000 Belgrade, Serbia.

ABSTRACT: In this paper we introduce *new* class of system, so called *singularly impulsive* or *generalized impulsive dynamical systems with time delay*. Dynamics of this system is characterized by the set of differential and difference equations with time delay, and algebraic equations. They represent the class of hybrid systems, where algebraic equations represent constraints that differential and difference equations with time delay need to satisfy. In this paper we present model, assumptions on the model, and two classes of singularly impulsive dynamical systems with delay - time dependent and state dependent. Further, we present Lyapunov - Krasovskii stability results for the class of singularly impulsive dynamical systems with time delay.

1 INTRODUCTION

Modern complex engineering systems as well as biological and physiological systems typically possess a multi-echelon hierarchical hybrid architecture characterized by continuous-time dynamics at the lower levels of hierarchy and discrete-time dynamics at the higher levels of the hierarchy. Hence, it is not surprising that hybrid systems have been the subject of intensive research over the past recent years (see Branicky et al. (1998), Ye et al. (1998 b), Haddad, Chellaboina and Kablar (2001a-b)). Such systems include dynamical switching systems Branicky (1998), Leonessa et al. (2000), nonsmooth impact and constrained mechanical systems, Back et al. (1993), Brogliato (1996), Brogliato et al. (1997), biological systems Lakshmikantham et al. (1989), demographic systems Liu (1994), sampled-data systems Hagiwara and Araki (1988), discrete-event systems Passino et al. (1994), intelligent vehicle/highway systems Lygeros et al. (1998) and flight control systems, etc. The mathematical descriptions of many of these systems can be characterized by impulsive differential equations, Simeonov and Bainov (1985), Liu (1988), Lakshmikantham et al. (1989, 1994), Bainov and Simeonov (1989, 1995), Kulev and Bainov (1989), Lakshmikantham and Liu (1989), Hu et al. (1989), Samoilenko and Perestyuk (1995), Haddad, Chellaboina and Kablar (2001a-b). Impulsive dynamical systems can be viewed as a subclass of hybrid systems.

Motivated by the results on impulsive dynamical systems presented in Haddad, Chellaboina, and Kablar (2001, 2005), the authors previous work on singular or generalized systems, and results on singularly impulsive dynamical systems published in Kablar(2003, 2010) we presented new class of *singularly impulsive* or *generalized impulsive dynamical systems with time delay*. It presents novel class of hybrid systems and generalization of impulsive dynamical systems to incorporate singular nature of the systems and time delays. Extensive applications of this class of systems can be found in contact problems and in hybrid systems.

We present mathematical model of the singularly impulsive dynamical systems with time delay. We show how it can be viewed as general systems from which impulsive dynamical systems with time delay, singular continuous-time systems with time delay and singular dicrete-time systems with time delay, as well as without time delay, follow. Then we present Assumptions needed for the model and the division of this class of systems to time-dependent and state-dependent singularly impulsive dynamical systems with time delay with respect to the resetting set.

In this paper for the class of nonlinear singularly impulsive dynamical systems with time delay we develop Lyapunov - Krasovskii and Razumikhin stability results. Results are further specialized to linear case. Note that for addressing the stability of the zero solution of a singularly impulsive dynamical system the usual stability definitions are valid. Finally, we draw some conclusions and define future work.

At first, we establish definitions and notations. Let $\mathbb{R}$ denote the set of real numbers, let $\mathbb{R}^n$ denote the set of $n \times 1$ real column vectors, let $\mathcal{N}$ denote the set of nonnegative integers, and let I_n or I denote the $n \times n$ identity matrix. Furthermore, let $\partial\mathcal{S}, \dot{\mathcal{S}}, \bar{\mathcal{S}}$ denote the boundary, the interior, and a closure of the subset $\mathcal{S} \subset \mathbb{R}^n$, respectively. Finally, let C^0 denote the set of continuous functions and C^{r} denote the set of functions with r continuous derivatives.

2 MATHEMATICAL MODEL OF SINGULARLY IMPULSIVE DYNAMICAL SYSTEMS WITH TIME DELAY

A singularly impulsive dynamical system with delay consists of three elements:

1. A possibly singular continuous-time dynamical equation with time delay, which governs the motion of the system between resetting events;

Received November 30, 2012 IFNA Publishers, Inc.

2. A possibly singular difference equation with time delay, which governs the way the states are instantaneously changed when a resetting occurs; and
3. A criterion for determining when the states of the system are to be reset.

Mathematical model of these systems is described with

$$E_{\rm c}\dot{x}(t) = f_{\rm c}(x(t,\tau)) + G_{\rm c}(x(t,\tau))u_{\rm c}(t), \quad (t, x(t,\tau), u_{\rm c}(t)) \notin \mathcal{S}, \tag{2.1}$$

$$E_{\rm d}\triangle x(t) = f_{\rm d}(x(t,\tau)) + G_{\rm d}(x(t,\tau))u_{\rm d}(t), \quad (t, x(t,\tau), u_{\rm c}(t)) \in \mathcal{S}, \tag{2.2}$$

$$y_{\rm c}(t) = h_{\rm c}(x(t,\tau)) + J_{\rm c}(x(t,\tau))u_{\rm c}(t), \quad (t, x(t,\tau), u_{\rm c}(t)) \notin \mathcal{S}, \tag{2.3}$$

$$y_{d}(t) = h_{\rm d}(x(t,\tau) + J_{\rm d}(x(t,\tau))u_{\rm d}(t), \quad (t, x(t,\tau), u_{\rm c}(t)) \in \mathcal{S}, \tag{2.4}$$

where $t \geq 0$, $\tau > 0$, $x(0) = x_0$, $x(t,\tau) \in \mathcal{D} \subset \mathbb{R}^n \times \mathbb{N}$, $\mathcal{D}$ is an open set with $0 \in \mathcal{D}$, $u_{\rm c} \in \mathcal{U}_{\rm c} \subset \mathbb{R}^{m_{\rm c}}$, $u_{\rm d}(t_k) \in \mathcal{U}_{\rm d} \subset \mathbb{R}^{m_{\rm d}}$, t_k denotes $k^{\rm th}$ instant of time at which $(t, x(t,\tau), u_{\rm c}(t))$ intersects $\mathcal{S}$ for a particular trajectory $x(t,\tau)$ and input $u_{\rm c}(t), y_{\rm c}(t) \in \mathbb{R}^{l_{\rm c}}, y_{\rm d}(t_k) \in \mathbb{R}^{l_{\rm d}}$, $f_{\rm c} : \mathcal{D} \to \mathbb{R}^n$ is Lipschitz continuous and satisfies $f_{\rm c}(0) = 0$, $G_{\rm c} : \mathcal{D} \to \ltimes \times \gtrdot_{\rm c}$, $f_{\rm d} : \mathcal{D} \to \mathbb{R}^{\ltimes}$ is continuous and satisfies $f_{\rm d}(0) = 0$, $G_{\rm d} : \mathcal{D} \to \mathbb{R}^{n\times m_{\rm d}}$, $h_{\rm c} : \mathcal{D} \to \mathbb{R}^{l_{\rm c}}$ and satisfies $h_{\rm c}(0) = 0$, $J_{\rm c} : \mathcal{D} \to \mathbb{R}^{l_{\rm c}\times m_{\rm c}}$, $h_{\rm d} : \mathcal{D} \to \mathbb{R}^{l_{\rm d}}$ and satisfies $h_{\rm d}(0) = 0$, $J_{\rm d} : \mathcal{D} \to \mathbb{R}^{l_{\rm d}\times m_{\rm d}}$, and $\mathcal{S} \subset [0,\infty) \times \mathbb{R}^n \times \mathcal{U}_{\rm c}$ is the *resetting set*. Here, as in Haddad, Chellaboina, and Kablar (2001a) we assume that $u_{\rm c}(\cdot)$ and $u_{\rm d}(\cdot)$ are restricted to the class of *admissible* inputs consisting of measurable functions $(u_{\rm c}(t), u_{\rm d}(t)) \in \mathcal{U}_{\rm c} \times \mathcal{U}_{\rm d}$ for all $t \geq 0$ and $k \in \mathcal{N}_{[0,t)} \equiv k : 0 \leq t_k < t$, where the constraint set $\mathcal{U}_{\rm c} \times \mathcal{U}_{\rm d}$ is given with $(0,0) \in \mathcal{U}_{\rm c} \times \mathcal{U}_{\rm d}$. We refer to the differential equation (2.1) as the *continuous-time dynamics with time delay*, and we refer to the difference equation (2.2) as the *resetting law*.

Matrices $E_{\rm c}$, $E_{\rm d}$ may be singular matrices. In case $E_{\rm c} = I$, $E_{\rm d} = I$, and $\tau = 0$ (2.1)–(2.4) represent standard impulsive dynamical systems described in Haddad, Chellaboina, and Kablar (2001a), and Haddad, Kablar, and Chellaboina (2000, 2005), where stability, dissipativity, feedback interconnections, optimality, robustness, and disturbance rejection has been analyzed. In absence of discrete dynamics they specialize to singular continuous-time systems, with further specialization $E_{\rm c} = I$ to standard continuous-time systems. If only discrete dynamics is present they specialize to singular discrete-time systems, with further specialization $E_{\rm d} = I$ to standard discrete-time systems.

In case $E_{\rm c} = I$, $E_{\rm d} = I$, and $\tau \neq 0$, (2.1)–(2.4) represent standard impulsive dynamical systems with time delay. In absence of discrete dynamics they specialize to singular continuous-time systems with time delay, with further specialization $E_{\rm c} = I$ to standard continuous-time systems with time delay. If only discrete dynamics is present they specialize to singular discrete-time systems with time delay, with further specialization $E_{\rm d} = I$ to standard discrete-time systems with time delay.

Therefore, theory of the singularly impulsive or generalized impulsive dynamical systems with time delay once developed, can be viewed as a generalization of the singular and impulsive dynamical system with time delay theory, unifying them into more general new system theory.

In what follows is given basic setting and division of this class of systems with respect to the definition of the resetting sets, accompanied with adequate assumptions needed for the model.

We make the following additional assumptions:

A1. $(0, x_0, u_{\rm c0}) \notin \mathcal{S}$, where $x(0) = x_0$ and $u_{\rm c}(0) = u_{\rm c0}$, that is, the initial condition is not in $\mathcal{S}$.

A2. If $(t, x(t,\tau), u_{\rm c}(t)) \in \bar{\mathcal{S}}\backslash\mathcal{S}$ then there exists $\epsilon > 0$ such that, for all $0 < \delta < \epsilon$, $s(t+\delta; t, x(t,\tau), u_{\rm c}(t+\delta)) \notin \mathcal{S}$.

A3. If $(t_k, x(t_k), u_{\rm c}(t_k)) \in \partial S \cap \mathcal{S}$ then there exists $\epsilon > 0$ such that, for all $0 < \delta < \epsilon$ and $u_{\rm d}(t_k) \in \mathcal{U}_{\rm d}$, $s(t_k + \delta; t_k, E_{\rm d}x(t_k) + f_{\rm d}(x(t_k)) + G_{\rm d}(x(t_k))u_{\rm d}(t_k), u_{\rm c}(t_k + \delta)) \notin \mathcal{S}$.

A4. We assume consistent initial conditions (and prior and after every resetting).

Assumption A1 ensures that the initial condition for the resetting differential equation (2.1), (2.2) is not a point of discontinuity, and this assumption is made for convenience. If $(0, x_0, u_{\rm c0}) \in \mathcal{S}$, then the system initially resets to $E_{\rm d}x_0^+ = E_{\rm d}x_0 + f_{\rm d}(x_0) + G_{\rm d}(x_0)u_{\rm d}(0)$ which serves as the initial condition for the continuous dynamics (2.1). It follows from A3 that the trajectory then leaves $\mathcal{S}$. We assume in A2 that if a trajectory reaches the closure of $\mathcal{S}$ at a point that does not belong to $\mathcal{S}$, then the trajectory must be directed away from $\mathcal{S}$, that is, a trajectory cannot enter $\mathcal{S}$ through a point that belongs to the closure of $\mathcal{S}$ but not to $\mathcal{S}$. Finally, A3 ensures that when a trajectory intersects the resetting set $\mathcal{S}$, it instantaneously exits $\mathcal{S}$, see Figure 1. We make the following remarks.

Figure 1. Resetting Set.

Remark 2.1. *It follows from A3 that resetting removes the pair $(t_k, x_k, u_{\rm c}(t_k))$ from the resetting set $\mathcal{S}$. Thus, immediately after resetting occurs, the continuous-time dynamics (2.1), and not the resetting law (2.2), becomes the active element of the singularly impulsive dynamical system.*

Remark 2.2. *It follows from A1-A3 that no trajectory can intersect the interior of $\mathcal{S}$. According to A1, the trajectory $x(t)$ begins outside the set $\mathcal{S}$. Furthermore, it follows from A2 that a trajectory can only reach $\mathcal{S}$ through a point belonging* *to both $\mathcal{S}$ and its boundary. Finally, from A3, it follows that if a trajectory reaches a point $\mathcal{S}$ that is on the boundary of $\mathcal{S}$, then the trajectory is instantaneously removed from $\mathcal{S}$. Since a continuous trajectory starting outside of $\mathcal{S}$ and intersecting the interior of $\mathcal{S}$ must first intersect the boundary of $\mathcal{S}$, it follows that no trajectory can reach the interior of $\mathcal{S}$.*

Remark 2.3. *It follows from A1-A3 and Remark 1.2 that $\partial\mathcal{S} \cup \mathcal{S}$ is closed and hence the resetting times t_k are well defined and distinct.*

Remark 2.4. *Since the resetting times are well defined and distinct, and since the solutions to (2.1) exist and are unique, it follows that the solutions of the singularly impulsive dynamical system (2.1), (2.2) also exist and are unique over a forward time interval.*

In Haddad, Chellaboina and Kablar (2001a), the resetting set $\mathcal{S}$ is defined in terms of a countable number of functions $n_k : \mathbb{R}^n \to (0, \infty)$, and is given by

$$\mathcal{S} = \cup_k \{(n_k(x), x, u_c(n_k(x)) : x \in \mathbb{R}^n\}. \tag{2.5}$$

The analysis of singularly impulsive dynamical systems with time delay and with a resetting set of the form (2.5) can be quite involved. In particular, such systems exhibit Zenoness, beating, as well as confluence phenomena wherein solutions exhibit infinitely many transitions in a finite times, and coincide after a given point of time, Haddad, Chellaboina and Kablar (2001a). In this paper we assume that existence and uniqueness properties of a given singularly impulsive dynamical system with time delay are satisfied in forward time. Furthermore, since singularly impulsive dynamical systems of the form (2.1)-(2.4) involve impulses at variable times they are time-varying systems.

Here we will consider singularly impulsive dynamical systems involving two distinct forms of the resetting set $\mathcal{S}$. In the first case, the resetting set is defined by a prescribed sequence of times which are independent of state x. These equations are thus called *time-dependent singularly impulsive dynamical systems with time delay*. In the second case, the resetting set is defined by a region in the state space that is independent of time. These equations are called *state-dependent singularly impulsive dynamical systems with time delay*.

2.1 Time-Dependent Singularly Impulsive Dynamical Systems with Time Delay

Time-dependent singularly impulsive dynamical systems with time delay can be written as (2.1)–(2.4) with $\mathcal{S}$ defined as

$$\mathcal{S} = n \times \mathbb{R}^n \times \mathcal{U}_c, \tag{2.6}$$

where

$$n = t_1, t_2, \ldots \tag{2.7}$$

and $0 < t_1 < t_2 < \ldots$ are prescribed resetting times. When an infinite number of resetting times are used and $t_k \to \infty$ as $k \to \infty$, then $\mathcal{S}$ is closed. Now (2.1)–(2.4) can be rewritten in the form of the *time-dependent singularly impulsive dynamical system with time delay*

$$E_c\dot{x}(t) = f_c(x(t,\tau)) + G_c(x(t,\tau))u_c(t),\ t \neq t_k, \tag{2.8}$$
$$E_d\triangle x(t) = f_d(x(t,\tau)) + G_d(x(t,\tau))u_d(t),\ t = t_k, \tag{2.9}$$
$$y_c(t) = h_c(x(t,\tau)) + J_c(x(t,\tau))u_c(t),\ t \neq t_k, \tag{2.10}$$
$$y_d(t) = h_d(x(t,\tau)) + J_d(x(t,\tau))u_d(t),\ t = t_k. \tag{2.11}$$

Since $0 \notin \tau$ and $t_k < t_{k+1}$, $\tau > 0$, it follows that the assumptions A1–A3 are satisfied. Since time-dependent singularly impulsive dynamical systems with time delay involve impulses at a fixed sequence of times, they are time-varying systems.

Remark 2.5. *Standard continuous-time and discrete-time dynamical systems as well as sampled-data systems can be treated as special cases of singularly impulsive dynamical systems. For details see [1].*

Remark 2.6. *The time-dependent singularly impulsive dynamical system with time delay (2.8)–(2.11), with $E_c = I$ and $E_d = I$ includes as a special case the impulsive control problem addressed in the literature wherein at least one of the state variables of the continuous-time plant can be changed instantaneously to any given value given by an impulsive control at a set of control instants τ, Haddad, Chellaboina and Kablar (2001a).*

2.2 State-Dependent Singularly Impulsive Dynamical Systems with Time Delay

State-dependent singularly impulsive dynamical systems with time delay can be written as (2.1)–(2.4) with $\mathcal{S}$ defined as

$$\mathcal{S} = [0, \infty) \times \mathcal{Z}, \tag{2.12}$$

where $\mathcal{Z} = \mathcal{Z}_x \times \mathcal{U}_\mathrm{c}$ and $\mathcal{Z}_x \subset \mathbb{R}^n$. Therefore, (2.1)–(2.4) can be rewritten in the form of the *state-dependent singularly impulsive dynamical system with time delay*

$$E_\mathrm{c}\dot{x}(t) = f_\mathrm{c}(x(t,\tau)) + G_\mathrm{c}(x(t,\tau))u_\mathrm{c}(t), \quad (x(t,\tau), u_\mathrm{c}(t)) \notin \mathcal{Z}, \tag{2.13}$$

$$E_\mathrm{d}\triangle x(t) = f_\mathrm{d}(x(t,\tau)) + G_\mathrm{d}(x(t,\tau))u_\mathrm{d}(t), \quad (x(t,\tau), u_\mathrm{c}(t)) \in \mathcal{Z}, \tag{2.14}$$

$$y_\mathrm{c}(t) = h_\mathrm{c}(x(t,\tau)) + J_\mathrm{c}(x(t,\tau))u_\mathrm{c}(t), \quad (x(t,\tau), u_\mathrm{c}(t)) \notin \mathcal{Z}, \tag{2.15}$$

$$y_d(t) = h_\mathrm{d}(x(t,\tau)) + J_\mathrm{d}(x(t,\tau))u_\mathrm{d}(t), \quad (x(t,\tau), u_\mathrm{c}(t)) \in \mathcal{Z}. \tag{2.16}$$

(2.17)

We assume that $(x_0, u_{\mathrm{c}0}) \notin \mathcal{Z}$, $\tau > 0$, $(0,0) \notin \mathcal{Z}$, and that the resetting action removes the pair (x, u_c) from the set $\mathcal{Z}$; that is, if $(x, u_\mathrm{c}) \in \mathcal{Z}$ then $(E_\mathrm{d}x + f_\mathrm{d}(x) + G_\mathrm{d}(x)u_\mathrm{d}, u_\mathrm{c}) \notin \mathcal{Z}$, $u_\mathrm{d} \in \mathcal{U}_\mathrm{d}$. In addition, we assume that if at time t the trajectory $(x(t,\tau), u_\mathrm{c}(t)) \in \bar{\mathcal{Z}} \backslash \mathcal{Z}$, then there exists $\epsilon > 0$ such that for $0 < \delta < \epsilon$, $(x(t+\tau+\delta), u_\mathrm{c}(t+\delta)) \notin \mathcal{Z}$.

These assumptions represent the specialization of A1–A3 for the particular resetting set (2.12). It follows from these assumptions that for a particular initial condition, the resetting times $\tau_k(x_0)$ are distinct and well defined. Since the resetting set $\mathcal{Z}$ is a subset of the state space and is independent of time, state-dependent singularly impulsive dynamical systems with time delay are time-invariant systems. Finally, in the case where $\mathcal{S} \equiv [0,\infty) \times \mathbb{R}^n \times \mathcal{Z}_{u_\mathrm{c}}$, where $\mathcal{Z}_{u_\mathrm{c}} \subset \mathcal{U}_\mathrm{c}$ we refer to (2.13)–(2.16) as an input-dependent singularly impulsive dynamical system with time delay. Both these cases represent a generalization to the impulsive control problem considered in the literature.

3 STABILITY OF SINGULARLY IMPULSIVE DYNAMICAL SYSTEMS WITH TIME DELAY: LYAPUNOV - KRASOVSKII STABILITY

We consider linear singular time-invariant systems with single delay

$$E_\mathrm{c}\dot{x}(t) = A_\mathrm{c}x(t) + A_{\mathrm{c}1}x(t-\tau), \quad (t, x(t,\tau)) \notin \mathcal{Z}, \tag{3.18}$$

$$E_\mathrm{d}x(k+1) = A_\mathrm{d}x(k) + A_{\mathrm{d}1}x(k-\tau), \quad (t, x(t,\tau)) \in \mathcal{Z}, \tag{3.19}$$

where $x \in \mathbb{R}^n$, A_c, A_d, $A_{\mathrm{c}1}$, $A_{\mathrm{d}1}$ are given $n \times n$ real matrices, and E_c, E_d may be singular matrices. The usual initial condition is in the form of

$$x_0(t) = \phi(t), \qquad (t, x(t,\tau)) \notin \mathcal{Z}, \tag{3.20}$$

for $t, k \in [-\tau, 0]$, where ϕ is a continuous function.

We will use Lyapunov-Krasovskii Stability Theorem to discuss the stability of the system. We will restrict ourselves to using Lyapunov-Krasovski functional, and aim at arriving at stability criteria that can be written in the form of Linear Matrix Inequalities (LMI) or a closely related form. Efficient numerical methods are available to solve LMI's.

We can state a restricted version of Lyapunov-Krasovskii Stability Theorem developed for the class of singularly impulsive dynamical systems.

Proposition 3.1. *A singularly impulsive dynamical system with time-delay system is asymptotically stable if there exists a bounded quadratic Lyapunov-Krasovskii functional $V(\phi)$ such that for some $\epsilon > 0$, it satisfies* Lyapunov-Krasovskii functional condition

$$V(\phi(t)) \geq \epsilon \|E_\mathrm{c}\phi(0)\|^2, \qquad (t, x(t,\tau)) \notin \mathcal{Z}, \tag{3.21}$$

$$V(\phi(k)) \geq \epsilon \|E_\mathrm{d}\phi(0)\|^2, \qquad (t, x(t,\tau)) \in \mathcal{Z}, \tag{3.22}$$

and its derivative along the system trajectory,

$$\dot{V}(\phi(t)) = \dot{V}(x(t))|_{x(t)=\phi(t)},\ (t, x(t,\tau)) \notin \mathcal{Z}, \tag{3.23}$$

$$\Delta V(\phi(k)) = \Delta V(x(k))|_{x(k)=\phi(k)},\ (t, x(t,\tau)) \in \mathcal{Z}, \tag{3.24}$$

satisfies Lyapunov-Krasovskii derivative condition

$$\dot{V}(x(t)) \leq -\epsilon \|E_\mathrm{c}\phi(0)\|^2, \qquad (t, x(t,\tau)) \notin \mathcal{Z}, \tag{3.25}$$

$$\delta V(x(k)) \leq -\epsilon \|E_\mathrm{d}\phi(k_0)\|^2, \qquad (t, x(t,\tau)) \in \mathcal{Z}, \tag{3.26}$$

4 DELAY-INDEPENDENT STABILITY CRITERIA BASED ON LYAPUNOV-KRASOVSKII STABILITY THEOREM

In this section, we will discuss the stability of the same system (3.18)-(3.19) using some simple Lyapunov-Krasovskii functional method. The results parallel those obtained by Razumikhin Theorem.

4.1 Systems with single delay

Consider again the system (3.18)-(3.19). Probably the simplest stability criterion van be obtained by using the following Lyapunov-Krasovskii functional

$$V(x(t)) = x^{\mathrm{T}}(t)E_{\mathrm{c}}^{\mathrm{T}}PE_{\mathrm{c}}x(t) + \int_{t-\tau}^{\tau} x^{\mathrm{T}}(\xi)S_{\mathrm{c}}x(\xi)\mathrm{d}\xi, \quad (t, x(t,\tau)) \notin \mathcal{Z}, \tag{4.27}$$

$$V(x(k)) = x^{\mathrm{T}}(t)E_{\mathrm{d}}^{\mathrm{T}}PE_{\mathrm{d}}x(k) + \sum_{t-\tau}^{\tau} x^{\mathrm{T}}(\xi)S_{\mathrm{d}}x(\xi), \quad (t, x(t,\tau)) \in \mathcal{Z} \tag{4.28}$$

where the matrices P and S_{c}, S_{d} are symmetric and positive definite.

It can be easily calculated that the derivative of V along the system trajectory is

$$\dot{V}(x(t)) = \begin{bmatrix} x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-\tau) \end{bmatrix} \cdot \begin{bmatrix} A_{\mathrm{c}}^{\mathrm{T}}PE_{\mathrm{c}} + E_{\mathrm{c}}^{\mathrm{T}}PA_{\mathrm{c}} + S_{\mathrm{c}} & E_{\mathrm{c}}^{\mathrm{T}}P \\ Ac & \\ A_{\mathrm{c}}^{\mathrm{T}}PE_{\mathrm{c}} & -S_{\mathrm{c}} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}, \tag{4.29}$$

and

$$\Delta V(x(k)) = \begin{bmatrix} x^{\mathrm{T}}(k) & x^{\mathrm{T}}(k-\tau) \end{bmatrix}) \cdot \begin{bmatrix} A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} - E_{\mathrm{d}}^{\mathrm{T}}PE_{\mathrm{d}} + S_{\mathrm{d}} & A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} \\ A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} & -S_{\mathrm{d}} + A_{\mathrm{d}}PA_{\mathrm{d}} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}), \tag{4.30}$$

or we can write x as ϕ to obtain

$$\dot{V}(x(t)) = \begin{pmatrix} \phi^{\mathrm{T}}(t) & \phi^{\mathrm{T}}(-\tau) \end{pmatrix} \cdot \begin{pmatrix} A_{\mathrm{c}}^{\mathrm{T}}PE_{\mathrm{c}} + E_{\mathrm{c}}^{\mathrm{T}}PA_{\mathrm{c}} + S_{\mathrm{c}} & E_{\mathrm{c}}^{\mathrm{T}}PA_{\mathrm{d}} \\ A_{\mathrm{c}}^{\mathrm{T}}PE_{\mathrm{c}} & -S_{\mathrm{c}} \end{pmatrix} \begin{pmatrix} \phi(t) \\ \phi(-\tau) \end{pmatrix}. \tag{4.31}$$

and

$$\dot{V}(x(k)) = \begin{bmatrix} \phi^{\mathrm{T}}(t) & \phi^{\mathrm{T}}(-\tau) \end{bmatrix} \cdot \begin{bmatrix} A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} - E_{\mathrm{d}}^{\mathrm{T}}PE_{\mathrm{d}} + S_{\mathrm{d}} & A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} \\ A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} & -S_{\mathrm{d}} + A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} \end{bmatrix} \begin{bmatrix} \phi(t) \\ \phi(-\tau) \end{bmatrix}). \tag{4.32}$$

It is clear that $\dot{V}(x) \leq -\epsilon\|E_{\mathrm{c}}x(t)\|^2$, and $\Delta V(k) \leq -\epsilon\|E_{\mathrm{d}}x(k)\|^2$ for some sufficiently small $\epsilon > 0$ if the matrix in the expression above is negative definite. Thus we can conclude the following.

Proposition 4.1. *System (3.18)-(3.19) is asymptotically stable if there exist real symmetric matrices*

$$E_{\mathrm{c}}^{\mathrm{T}}PE_{\mathrm{c}} > 0 \tag{4.33}$$

$$E_{\mathrm{d}}^{\mathrm{T}}PE_{\mathrm{d}} > 0 \tag{4.34}$$

and $S_{\mathrm{c}}, S_{\mathrm{d}}$ such that

$$\begin{pmatrix} A_{\mathrm{c}}^{\mathrm{T}}PE_{\mathrm{c}} + E_{\mathrm{c}}^{\mathrm{T}}PA_{\mathrm{c}} + S & E_{\mathrm{c}}^{\mathrm{T}}PA_{\mathrm{d}} \\ A_{\mathrm{d}}^{\mathrm{T}}PE_{\mathrm{c}} & -S \end{pmatrix} < 0 \tag{4.35}$$

$$\begin{pmatrix} A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} - E_{\mathrm{d}}^{\mathrm{T}}PE_{\mathrm{d}} + S_{\mathrm{d}} & A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} \\ A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} & -S_{\mathrm{d}} + A_{\mathrm{d}}^{\mathrm{T}}PA_{\mathrm{d}} \end{pmatrix} < 0 \tag{4.36}$$

are satisfied.

Proof. Use Proposition 4.1, and choose Lyapunov-Krasovskii functional 4.27–4.28. Notice that 4.35–4.36 implies

$$S_{\mathrm{c}} > 0, \tag{4.37}$$
$$S_{\mathrm{d}} > 0, \tag{4.38}$$

which together with 4.33–4.34 implies

$$V(\phi) \geq \epsilon \|E_{\mathrm{c}}\phi(0)\|^2, \tag{4.39}$$
$$V(\phi) \geq \epsilon \|E_{\mathrm{d}}\phi(0)\|^2, \tag{4.40}$$

for some sufficiently small $\epsilon > 0$. The Lyapunov-Krasovskii functional condition is satisfied. Also, (**??**) implies that

$$\dot{V}(\phi) \geq \epsilon \|E_{\mathrm{c}}\phi(0)\|^2, \tag{4.41}$$
$$\Delta V(\phi) \geq \epsilon \|E_{\mathrm{d}}\phi(0)\|^2 \tag{4.42}$$

in view of 4.31–4.32 , and the Lyapunov-Krasovskii derivative condition is also satisfied. Therefore, the system is asymptotically stable according to Proposition 4.1. □

It is interesting to compare the stability criterion in Proposition 4.1 obtained using the Lyapunov-Krasovskii Stability Theorem with the corresponding criterion in Proposition 4 of Kablar (2012), obtained using the Razumikhin Theorem. It can be seen that the conditions in Proposition 4 can be obtained from those of Proposition 4.1 by introducing the additional constraint

$$S_{\mathrm{c}} = \alpha E_{\mathrm{c}}^{\mathrm{T}} P E_{\mathrm{c}}, \tag{4.43}$$
$$S_{\mathrm{d}} = \alpha E_{\mathrm{c}}^{\mathrm{T}} P E_{\mathrm{c}}, \tag{4.44}$$

4.2 Systems with distributed delay

Consider the system with distributed delays

$$E_{\mathrm{c}}\dot{x}(t) = A_{\mathrm{c}}x(t) + \int_{-\tau}^{0} A_{\mathrm{c1}}(\theta)x(t+\theta)\mathrm{d}\theta, \tag{4.45}$$

$$E_{\mathrm{d}}\dot{x}(k) = A_{\mathrm{d}}x(k) + \sum_{-\tau}^{0} A_{\mathrm{d1}}(\theta)x(k+\theta)\mathrm{d}\theta, \tag{4.46}$$

where $A_{\mathrm{c}}, A_{\mathrm{d}}$ are given constant matrices and $A_{\mathrm{c1}}(\theta), A_{\mathrm{d1}}(\theta)$ are continuous matrix valued function given for $\theta \in [-\tau, 0]$.

The Lyapunov-Krasovskii functional for the system can be chosen as

$$V(\phi) = \phi^{\mathrm{T}}(t)P\phi(t) + \int_{-\tau}^{0} [\int_{\theta}^{0} \phi^{\mathrm{T}}(\xi)S_{\mathrm{c}}(\theta)\phi(\xi)\mathrm{d}\xi]\mathrm{d}\theta, \tag{4.47}$$

$$V(\phi) = \phi^{\mathrm{T}}(t)P\phi(t) + \sum_{-\tau}^{0} [\sum_{\theta}^{0} \phi^{\mathrm{T}}(\xi)S_{\mathrm{d}}(\theta)\phi(\xi)], \tag{4.48}$$

$$\tag{4.49}$$

The derivative of $V(x)$ can be calculated as

$$\begin{aligned}\dot{V}(\phi) &= \phi^{\mathrm{T}}(0)[E_{\mathrm{c}}^{\mathrm{T}} P A_{\mathrm{c}} + A_{\mathrm{c}}^{\mathrm{T}} P E_{\mathrm{c}} + \int_{-\tau}^{0} S_{\mathrm{c}}(\theta)\mathrm{d}\theta] \\ &= 2\phi^{\mathrm{T}}(0)\int_{-\tau}^{0} E_{\mathrm{c}}^{\mathrm{T}} P A_{\mathrm{d}}(\theta)\phi(\theta)\mathrm{d}\theta \\ &\quad - \int_{-\tau}^{0} \phi^{\mathrm{T}}(\theta)S_{\mathrm{c}}(\theta)\phi(\theta)\mathrm{d}\theta,\end{aligned} \tag{4.50}$$

and

$$\begin{aligned}\Delta V(\phi) &= \phi^{\mathrm{T}}(k_0)[A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}} - E_{\mathrm{d}}^{\mathrm{T}} P E_{\mathrm{d}} + \sum_{-\tau}^{0} S_{\mathrm{d}}(\theta)\mathrm{d}\theta] \\ &= 2\phi^{\mathrm{T}}(0)\sum_{-\tau}^{0} A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}}(\theta)(\theta)\mathrm{d}\theta \\ &\quad - \sum_{-\tau}^{0} \phi^{\mathrm{T}}(\theta)S_{\mathrm{d}}(\theta)\phi(\theta)\mathrm{d}\theta,\end{aligned} \tag{4.51}$$

To facilitate further development, add and subtract a term involving the relaxation matrix function $R_{\rm c}(\theta), R_{\rm d}(\theta)$ in the above, resulting in

$$\begin{aligned}\dot{V}(\phi) &= \phi^{\rm T}(0)[E_{\rm c}^{\rm T}PA_{\rm c} + A_{\rm c}^{\rm T}PE_{\rm c} + \int_{-\tau}^{0} R(\theta){\rm d}\theta]\phi(0) \\ &+ \int_{-\tau}^{0} \begin{pmatrix} \phi^{\rm T}(0) & \phi^{\rm T}(\theta) \end{pmatrix} \\ &\cdot \begin{pmatrix} S_{\rm c}(\theta)R_{\rm c}(\theta) & E_{\rm c}^{\rm T}PA_{\rm d}(\theta) \\ A_{\rm d}^{\rm T}(\theta)PE_{\rm c} & -S_{\rm c}(\theta) \end{pmatrix}\begin{pmatrix} \phi(0) \\ \phi(\theta) \end{pmatrix}. \end{aligned} \tag{4.52}$$

and

$$\begin{aligned}\Delta V(\phi) &= \phi^{\rm T}(k_0)[A_{\rm d}^{\rm T}PA_{\rm d} - E_{\rm d}^{\rm T}PE_{\rm d} + \sum_{-\tau}^{0} R_{\rm d}(\theta){\rm d}\theta]\phi(k_0) \\ &+ \sum_{-\tau}^{0} \begin{pmatrix} \phi^{\rm T}(k_0) & \phi^{\rm T}(\theta) \end{pmatrix} \\ &\cdot \begin{pmatrix} S(\theta) - R_{\rm d}(\theta) & A_{\rm d}^{\rm T}PA_{\rm d}(\theta) \\ A_{\rm d}^{\rm T}(\theta)PA_{\rm d} & -S_{\rm d}(\theta) + A_{\rm d}^{\rm T}PA_{\rm d} \end{pmatrix}\begin{pmatrix} \phi(k_0) \\ \phi(\theta) \end{pmatrix}. \end{aligned} \tag{4.53}$$

The derivative condition will be satisfied if the two matrices in the above are negative definite.

Proposition 4.2. *The system described by (??) is asymptotically stable if there exists real symmetric matrix*

$$E_{\rm c}^{\rm T}PE_{\rm c} > 0 \tag{4.54}$$

$$E_{\rm d}^{\rm T}PE_{\rm d} > 0 \tag{4.55}$$

$$\tag{4.56}$$

and real symmetric matrix functions $R_{\rm c}(\theta), R_{\rm d}(\theta)$ *and* $S_{\rm c}(\theta), S_{\rm d}(\theta)$*, such that*

$$E_{\rm c}^{\rm T}PA_{\rm c} + A_{\rm c}^{\rm T}PE_{\rm c} + \int_{-tau}^{0} R_{\rm c}(\theta){\rm d}\theta < 0 \tag{4.57}$$

$$A_{\rm d}^{\rm T}PA_{\rm d} - E_{\rm d}^{\rm T}PE_{\rm d} + \sum_{-tau}^{0} R_{\rm d}(\theta) < 0 \tag{4.58}$$

and

$$\begin{pmatrix} S(\theta) - R(\theta & E_{\rm c}^{\rm T}PA_{\rm d}(\theta) \\ A_{\rm d}^{\rm T}(\theta)PE_{\rm c} & -S(\theta) \end{pmatrix} < 0, \quad \theta \in [-\tau, 0]. \tag{4.59}$$

and

$$\begin{pmatrix} S_{\rm d}(\theta) - R_{\rm d}(\theta & A_{\rm d}^{\rm T}PA_{\rm d}(\theta) \\ A_{\rm d}^{\rm T}(\theta)PA_{\rm d} & -S_{\rm d}(\theta) + A_{\rm d}^{\rm T}PA_{\rm d} \end{pmatrix} < 0, \quad \theta \in [-\tau, 0]. \tag{4.60}$$

Proof. Use Proposition 5.2 and Lyapunov-Krasovskii functional (4.47)–(4.48). Since (4.59)–(4.60) implies $S_{\rm c}(\theta) > 0, S_{\rm d}(\theta) > 0$, it is clear that $V(\phi) \geq \epsilon\|E_{\rm c}\phi(0)\|^2$ and $V(\phi) \geq \epsilon\|E_{\rm d}\phi(0)\|^2$ for some sufficiently small $\epsilon > 0$, the Lyapunov-Krasovskii functional condition (4.27)–(4.28) are satisfied. In expression (4.52)–(4.53), the second term is always less then or equal the zero due to (4.59)–(4.60). Also (4.57)–(4.58) implies the existence of a sufficiently small $\epsilon > 0$ such that

$$E_{\rm c}^{\rm T}PA_{\rm c} + A_{\rm c}^{\rm T}PE_{\rm c} + \int_{-tau}^{0} R_{\rm c}(\theta){\rm d}\theta < -\epsilon I, \tag{4.61}$$

$$A_{\rm d}^{\rm T}PA_{\rm d} - E_{\rm d}^{\rm T}PE_{\rm d} + \sum_{-tau}^{0} R_{\rm d}(\theta){\rm d}\theta < -\epsilon I, \tag{4.62}$$

$$\tag{4.63}$$

Therefore, $\dot{V}(\phi) \leq -\epsilon\|E_{\rm c}\phi(0)\|^2$, $\delta V(\phi) \leq -\epsilon\|E_{\rm d}\phi(0)\|^2$, and the Lyapunov-Krasovskii derivative condition are also satisfied. Therefore, the system is asymptotically stable according to Proposition 4.1. □

We can show that the above stability criterion is delay-independent. Also, the corresponding result using the Razumikhin Theorem (Proposition 4 of Kablar (2012) can be obtained from the above result by introducing additional constraint

$$S_{\rm c}(\theta) = \alpha(\theta)E_{\rm c}^{\rm T}PE_{\rm c}, \tag{4.64}$$

$$S_{\rm d}(\theta) = \alpha(\theta)E_{\rm d}^{\rm T}PE_{\rm d}, \tag{4.65}$$

5 DELAY-DEPENDENT STABILITY CRITERIA BASED ON LYAPUNOV-KRASOVSKII STABILITY THEOREM

Simple delay-dependent stability criterion can also be derived with the Lyapunov-Krasovskii Theorem, in parallel to Razumikhin Theorem formulation of Kablar (2012). Consider a system described by (3.18)–(3.19). Recall that we can use model transformation to obtain a system with distributed delays represented by (**??**) to (**??**). The stability of the system described by (**??**) to (**??**) implies that of (3.18)–(3.19). We can apply Proposition 4.1 to obtain the following Proposition.

Proposition 5.1. *The system described by (3.18)–(3.19) is asymptotically stable if there exist real symmetric matrices* $P, R_{\mathrm{c}0}, R_{\mathrm{d}0}, R_{\mathrm{c}1}, R_{\mathrm{d}1}, S_{\mathrm{c}0}, S_{\mathrm{d}0}, and S_{\mathrm{c}1}, and S_{\mathrm{d}1}$ *such that*

$$E_{\mathrm{c}}^{\mathrm{T}} P E_{\mathrm{c}} > 0 \tag{5.66}$$

$$\begin{pmatrix} \mathcal{M}_{\mathrm{c}} & -E_{\mathrm{c}}^{\mathrm{T}} P A_{\mathrm{c}1} A_{\mathrm{c}} & -E_{\mathrm{c}}^{\mathrm{T}} P A_{\mathrm{c}1}^2 \\ -A_{\mathrm{c}}^{\mathrm{T}} A_{\mathrm{c}1}^{\mathrm{T}} P E_{\mathrm{c}} & -S_{\mathrm{c}0} & 0 \\ -(A_{\mathrm{c}1}^2)^{\mathrm{T}} P E_{\mathrm{c}} & 0 & -S_{\mathrm{c}1} \end{pmatrix} < 0 \tag{5.67}$$

and

$$E_{\mathrm{d}}^{\mathrm{T}} P E_{\mathrm{d}} > 0 \tag{5.68}$$

$$\begin{pmatrix} \mathcal{M}_{\mathrm{d}} & -A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}1} A_{\mathrm{d}} & -A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}1}^2 \\ -A_{\mathrm{d}}^{\mathrm{T}} A_{\mathrm{d}1}^{\mathrm{T}} P A_{\mathrm{d}} & -S_{\mathrm{d}0} & 0 \\ -(A_{\mathrm{d}1}^2)^{\mathrm{T}} P A_{\mathrm{d}} & 0 & -S_{\mathrm{d}1} + A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}} \end{pmatrix} < 0 \tag{5.69}$$

where

$$\mathcal{M}_{\mathrm{c}} = \frac{1}{\tau}[E_{\mathrm{c}}^{\mathrm{T}} P(A_{\mathrm{c}} + A_{\mathrm{c}1}) + (A_{\mathrm{c}} + A_{\mathrm{c}1})^{\mathrm{T}} P E_{\mathrm{c}}] + S_{\mathrm{c}0} + S_{\mathrm{c}1}. \tag{5.70}$$

$$\mathcal{M}_{\mathrm{d}} = \frac{1}{\tau}[A_{\mathrm{d}}^{\mathrm{T}} P(A_{\mathrm{d}} + A_{\mathrm{d}1}) + (A_{\mathrm{d}} + A_{\mathrm{d}1})^{\mathrm{T}} P E_{\mathrm{c}}] + S_{\mathrm{d}0} + S_{\mathrm{d}1}. \tag{5.71}$$

Proof. As is discussed above, it is sufficient to prove the stability of the transformed system described by (**??**)–(**??**) and (**??**)–(**??**). Apply Proposition 4.1 and choose

$$R_{\mathrm{c}}(\theta) = \begin{cases} R_{\mathrm{c}0} & -\tau \le \theta < 0, \\ R_{\mathrm{c}1} & -2\tau \le \theta < -\tau, \end{cases} \tag{5.72}$$

$$Sd(\theta) = \begin{cases} S_{\mathrm{c}0} & -\tau \le \theta < 0, \\ S_{\mathrm{c}1} & -2\tau \le \theta < -\tau, \end{cases} \tag{5.73}$$

and

$$R_{\mathrm{d}}(\theta) = \begin{cases} R_{\mathrm{d}0} & -\tau \le \theta < 0, \\ R_{\mathrm{d}1} & -2\tau \le \theta < -\tau, \end{cases} \tag{5.74}$$

$$S_{\mathrm{d}}(\theta) = \begin{cases} S_{\mathrm{d}0} & -\tau \le \theta < 0, \\ S_{\mathrm{d}1} & -2\tau \le \theta < -\tau, \end{cases} \tag{5.75}$$

to obtain the sufficient condition consisting of (5.66)–(5.68) and

$$[E_{\mathrm{c}}^{\mathrm{T}} P(A_{\mathrm{c}} + A_{\mathrm{c}1}) + (A_{\mathrm{c}} + A_{\mathrm{c}1})^{\mathrm{T}} P E_{\mathrm{c}}] + \tau(R_{\mathrm{c}0} + R_{\mathrm{c}1}) < 0, \tag{5.76}$$

$$\begin{pmatrix} S_{\mathrm{c}k} - R_{\mathrm{c}k} & -E_{\mathrm{c}}^{\mathrm{T}} P A_{\mathrm{c}1} A_{\mathrm{c}k} \\ A_{\mathrm{c}k}^{\mathrm{T}} A_{;c1}^{\mathrm{T}} P E_{\mathrm{c}} & -S_{\mathrm{c}k} \end{pmatrix} < 0, k = 0, 1. \tag{5.77}$$

and

$$[A_{\mathrm{d}}^{\mathrm{T}} P(A_{\mathrm{d}} + A_{\mathrm{d}1) + (\mathrm{A_d + A_{d1}})^{\mathrm{T}} \mathrm{PA_d}] + \tau(\mathrm{R_{d0} + R_{d1}}) < 0,} \tag{5.78}$$

$$\begin{pmatrix} S_{\mathrm{d}k} - R_{\mathrm{d}k} & -A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}1} A_{\mathrm{d}k} \\ A_{\mathrm{d}k}^{\mathrm{T}} A_{\mathrm{d}1}^{\mathrm{T}} P & \\ Ad & -S_{\mathrm{d}k} + A_{\mathrm{d}}^{\mathrm{T}} P A_{\mathrm{d}} \end{pmatrix} < 0, k = 0, 1. \tag{5.79}$$

Divide (5.77)–(5.79) by τ, and eliminate $R_{\mathrm{c}0}, R_{\mathrm{d}0}$ and $R_{\mathrm{c}1}, R_{\mathrm{d}1}$ from the resulting matrix inequality and (5.77)–(5.79) to obtain (5.67)–(5.69). □

Once again, the corresponding result in Proposition 5.1 can be obtained by applying additional constraints

$$S_{\mathrm{c}k} = \alpha_k P, \tag{5.80}$$

$$S_{\mathrm{d}k} = \alpha_k P, \tag{5.81}$$

(5.82)

If one is to consider the stability of only the system (3.18)–(3.19), the above criterion is clearly better that Proposition 5.1 since it is less conservative and computationally more convenient because of the linearity of parameters. The main value of Proposition 5.1 is again it is also valid for time-varying delay, as will be discussed in future work of robust stability analyse of singularly impulsive dynamical systems with time delay.

6 CONSLUSION

In this paper we have presented new class of singularly impulsive dynamical systems with time delay. For this class systems we have developed asymptotic stability results by using Lyapunov - Krasovskii stability theorem.

7 FUTURE WORK

In future work we will develop stability results based on Razumikhin stabity theorem, and robust stability results for the class of singularly impulsive dynamical systems based on Razumikhin and Lyapunov - Krasovskii stability theorem.

REFERENCES

[1] Dragutin Lj. Debeljovic, *Linear Singular Systems with pure Time Delay*, Faculty of Mechanical , University of Belgrade, 2010.

[2] D. Lj. Debeljkovic and N. A. Kablar, ”Finite Time Stability of linear singular systems: Bellman-Gronwall Approach”, *Proceedings of the American Control Conference*, San Diego, California, 1999, pp. 1803–1806.

[3] Haddad W.M., V.Chellaboina, N.A. Kablar, ”Nonlinear Impulsive Dynamical Systems: Stability and Dissipativity,” *Proc. IEEE Conf. Dec. Contr.*, pp. 5158-5163, Phoenix, AZ, 1999. Also in: *Int. J. Contr.*, vol. 74, pp. 1631-1658, 2001a.

[4] Haddad W.M., V.Chellaboina, N.A. Kablar, ”Nonlinear Impulsive Dynamical Systems: Feedback Interconnections and Optimality,” *Proc. IEEE Conf. Dec. Contr.*, Phoenix, AZ, 1999. Also in: *Int. J. Contr.*, vol. 74, pp. 1659-1677, 2001b.

[5] Kablar N.A., ”Singularly Impulsive or Generalized Impulsive Dynamical Systems,” *Proc. Amer. Contr. Conf.*, Denver, CO, 2003a.

[6] Kablar N.A., ”Singularly Impulsive Dynamical Systems: Lyapunov and Asymptotic Stability,” *Proc. Conf. Dec. Contr.*, USA, 2003b.

[7] Kablar N. A., and D. Lj. Debeljkovic, ”Finite time instability of time varying linear singular systems”, *Proceedings of the American Control Conference*, San Diego, California, 1999, pp. 1796-1800.

[8] Kablar N. A., and D. Lj. Debeljkovic, ”Finite time stability of time varying linear singular systems”, *Proceedings of the 37th IEEE Conference on Decision and Control*, Tampa, Florida, USA, 1998, pp. 3831–3836.

[9] Gu K. V., L. Kharitonov and J. Chen, *Stability of Time Delay Systems*, Birkhauser, Boston, 2003.