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Parametric Survival Analysis: A Comparison of Prostate Cancer Survivorship by Race

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Abstract: The objective of the present study is to perform parametric survival analysis to compare the survivorship of prostate cancer patients by race. In our analysis, we will compare the survival times between White and African American men under similar treatment in the four different stages of prostate cancer. In addition, we perform survival probability residual analysis to evaluate the average difference in survivorship between White and African American men.

1 INTRODUCTION

Prostate cancer is the third most common cause of death from cancer in men of all ages especially over age 75 [1]. According to the National Cancer Institute at the National Institutes of Health, the estimated new cases of prostate cancer in the United States in 2012 are 241,740. There were 28,170 deaths, which is approximately 11.7% [2]. Researchers had reported several risk factors related to prostate cancer [3-4]. Some of the risk factors include: age (> 60 years), family history of prostate cancer, abuse of alcohol, and an overweight of body mass index (BMI). The early stage of prostate cancer usually has no symptoms. However, based on the patient's level of risk, most physicians recommend going through the screening test. The most common methods of screening are Prostate-Specific Antigen (PSA) test, urine test, and digital test. Due to discrepancies in the methods of screening, prostate biopsy is the only test that can confirm the diagnosis [1].

Hosain, G. et al. (2012), in a study conducted to explore racial differences in treatment discussed, preferred and ultimately received for localized prostate cancer, has revealed that health professionals were more likely to discuss surgery and watchful waiting with Whites than Hispanics. African Americans on the other hand, were less likely to receive watchful waiting. However, they preferred and received radiation therapy. In general, patients who preferred surgery and radiation therapy showed a higher agreement for what was actually received [5].

Pulte, D. et al. (2012) in their paper examined changes in prostate cancer survival by ethnicity. Results showed that there was an improvement in African Americans and Hispanics than non-Hispanic whites. There has been a decrease in survival disparities between non-Hispanic white and minorities [6].

In this study, our goal is to compare the survivorship between White and African American men at the four different stages of prostate cancer under similar treatment. In addition, the average difference in survival between Whites and African Americans would be estimated.

2 DATA DESCRIPTION

Data on the survival time of 23,714 Adenocarcinoma prostate cancer patients for Whites and African Americans were obtained from the Surveillance Epidemiology and End Results (SEER) program. Out of 23,714, approximately 91.6% are Whites and 8.4% are African Americans. Other variables considered are type of treatment and stage of prostate cancer. 'Treatment' is categorized as radiation only (Rad), surgery only (Surg), combination of radiation and surgery (RadSurg), and no treatment (NoTreat). The stage of prostate cancer is coded 'I' for stage 1, 'II' for stage 2, 'III' for stage 3 and 'IV' for stage 4. 'Censor' is coded '0' if the patient died before the end of the study and '1' if the patient survived at the end of the study. It is worth mentioning that all recorded deaths in this study were due to prostate cancer. An extended detail of the dataset is given by Figure 2.1.

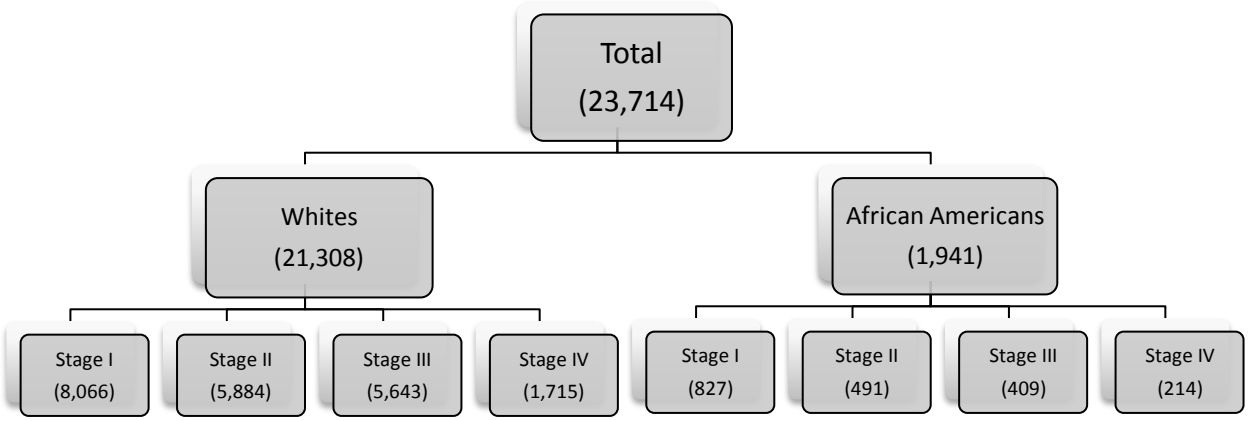

Figure 2.1: Schematic Diagram of Prostate Cancer Patients by Race and the Stage of the Cancer.

The survival times were calculated using the date of diagnosis and one of the following: date of death (i.e. if patient died before the end of the study), follow-up cutoff date (i.e. if the patient survived at the end of the study), or date last known to be alive. The follow-up cutoff date used in this study was December 31, 2008.

3 PAREMETRIC SURVIVAL ANALYSIS

In order to perform the parametric survival analysis, we need to identify appropriate probability distribution function that characterizes the behavior of the survival times. Different classical distributions were fitted to the observed data. The observed survival times follow a two parameter gamma distribution.

The two parameter gamma probability distribution function with shape parameter κ and location parameter λ is given by

$$f(t,\kappa,\lambda) = \frac{\lambda(\lambda t)^{\kappa-1}}{\Gamma(\kappa)}\exp(-\lambda t)\,, where\ \kappa,\lambda > 0, t \geq 0, and\ \Gamma(\kappa) = \int_0^\infty t^{\kappa-1}e^{-t}dt$$

The corresponding survival function is given by

$$S(t) = 1 - F(t) = 1 - \int_0^t f(x)dx = 1 - \frac{\Gamma_t\left(\kappa,\frac{t}{\lambda}\right)}{\Gamma(\kappa)} \quad (1)$$

where $\Gamma_t(\kappa, t) = \int_0^t t^{\kappa-1}e^{-t}dt$ is the lower incomplete gamma function and the hazard function given by

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{t^{\kappa-1}e^{-t}}{\Gamma(\kappa)-\Gamma_t(\kappa)}$$

$where\ F(t)$ is the cumulative density function of $f(t)$. The survival function has no closed-form expression. However, there exist algorithms for its computation.

3.1 Comparison and Evaluation of Survivorship by Race

The best fitted probability distribution function that characterizes the behavior of the survival times for White men, African American men, and both races is the gamma distribution. The approximate maximum likelihood estimates (MLE) and the corresponding estimated parametric survival and hazard functions for White men, African American men, and both races are shown in Table 3.1.1.

A graphical display of the estimated survival functions are given by Figure 3.1.1. The graphs reveal that the estimated survival function for African American men is different from the combine race. However, the estimated survival function for White men appears to be approximately the same as the combined races.

Table 3.1.1: Maximum Likelihood Estimates, Survival Functions, and Hazard Functions for All Stages by Race

Race	MLE	Estimated Survival Function	Estimated Hazard Function
Both	$\hat{\kappa} = 4.0476$ $\hat{\lambda} = 2.4244$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.0476, \frac{t}{2.4244}\right)}{\Gamma(4.0476)}$	$\hat{h}(t) = \frac{t^{4.0476-1}e^{-t}}{\Gamma(4.0476) - \Gamma_t(4.0476)}$
White	$\hat{\kappa} = 4.1216$ $\hat{\lambda} = 2.397$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.1216, \frac{t}{2.397}\right)}{\Gamma(4.1216)}$	$\hat{h}(t) = \frac{t^{4.1216-1}e^{-t}}{\Gamma(4.1216) - \Gamma_t(4.1216)}$
African American	$\hat{\kappa} = 3.3782$ $\hat{\lambda} = 2.6883$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.3782, \frac{t}{2.6883}\right)}{\Gamma(3.3782)}$	$\hat{h}(t) = \frac{t^{3.3782-1}e^{-t}}{\Gamma(3.3782) - \Gamma_t(3.3782)}$

Based on these findings, we proceed to perform survival probability residual analysis by discretizing the time points. The difference between two survival curves at each time point is defined as the survival probability residuals. Let $\hat{\varepsilon}_{CW}$ represent the survival probability residual between combined races and White. That is

$$\hat{\varepsilon}_{CW} = \hat{S}_{Combine}(t_i) - \hat{S}_{White}(t_i), i = 1, 2, 3, \cdots, 21$$

While $\hat{\varepsilon}_{CAA}$ represent the survival probability residual between combined races and African American. That is

$$\hat{\varepsilon}_{CAA} = \hat{S}_{Combine}(t_i) - \hat{S}_{African\ American}(t_i), i = 1, 2, 3, \cdots, 21$$

The estimated mean probability residual between combined races and African Americans is approximately 0.0347 and between combined races and Whites is approximately -0.0032. This indicates that making a decision for African Americans using the combined races is overestimated by approximately 3.47% while Whites is underestimated by approximately 0.32%. Furthermore, the average mean tumor for white men is statistically larger than African-American men [7]. Thus, it is essential to consider the survivorship of Whites and African Americans separately for efficient decision making.

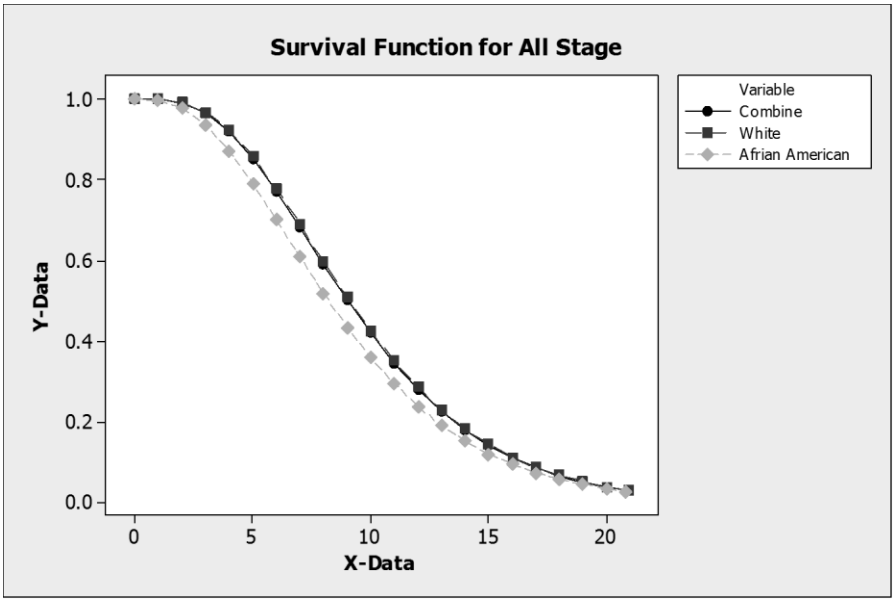

Figure 3.1.1: Survival Function for All Stages by Race

3.2 Comparison and Evaluation of Survivorship by Stage

The best fitted probability distribution function that characterizes the behavior of the survival times for stage I, II, III, and IV is the gamma distribution. The approximate maximum likelihood estimates (MLE) and the corresponding estimated parametric survival and hazard functions for Stage I, II, III, and IV are shown in Table 3.2.1.

A graphical display of the estimated survival functions is given by Figure 3.2.1. The graphs reveal that the estimated survival function for stage I, II, III, and IV are different compared with all stage. The estimated mean probabilities of the residuals for stage I, II, III, and IV compared with all stages are approximately 2.37%, -1.07%, -5.79%, and 10.17%, respectively. This reveals that making a decision using the combined stages is overestimated for stage I and IV while is underestimated for stage II and III. Thus, it is essential to identify the stage of the prostate cancer in order to make an efficient decision.

Table 3.2.1: Maximum Likelihood Estimates, Survival Functions, and Hazard Functions for both Races by Stage

Stage	MLE	Estimated Survival Function	Estimated Hazard Function
All Stage	$\hat{\kappa} = 4.0476$ $\hat{\lambda} = 2.4244$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.0476, \frac{t}{2.4244}\right)}{\Gamma(4.0476)}$	$\hat{h}(t) = \frac{t^{4.0476-1}e^{-t}}{\Gamma(4.0476) - \Gamma_t(4.0476)}$

I	$\hat{\kappa} = 3.6238$ $\hat{\lambda} = 2.5578$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.6238, \frac{t}{2.5578}\right)}{\Gamma(3.6238)}$	$\hat{h}(t) = \frac{t^{3.6238-1}e^{-t}}{\Gamma(3.6238) - \Gamma_t(3.6238)}$
II	$\hat{\kappa} = 4.3383$ $\hat{\lambda} = 2.3173$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.3383, \frac{t}{2.3173}\right)}{\Gamma(4.3383)}$	$\hat{h}(t) = \frac{t^{4.3383-1}e^{-t}}{\Gamma(4.3383) - \Gamma_t(4.3383)}$
III	$\hat{\kappa} = 5.7695$ $\hat{\lambda} = 1.9191$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(5.7695, \frac{t}{1.9191}\right)}{\Gamma(5.7695)}$	$\hat{h}(t) = \frac{t^{5.7695-1}e^{-t}}{\Gamma(5.7695) - \Gamma_t(5.7695)}$
IV	$\hat{\kappa} = 2.5843$ $\hat{\lambda} = 2.9435$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(2.5843, \frac{t}{2.9435}\right)}{\Gamma(2.5843)}$	$\hat{h}(t) = \frac{t^{2.5843-1}e^{-t}}{\Gamma(2.5843) - \Gamma_t(2.5843)}$

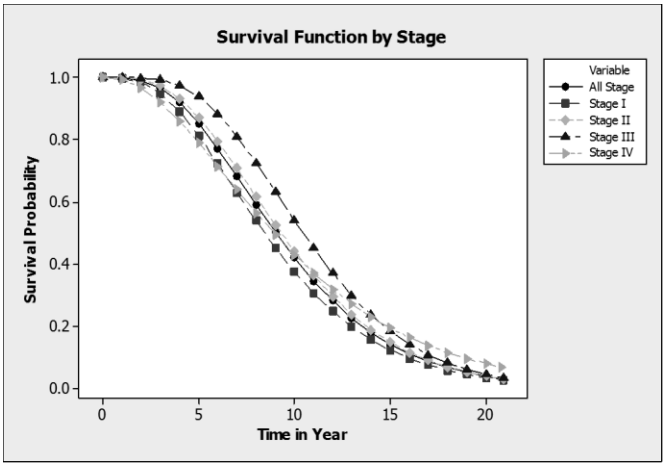

Figure 3.2.1: Survival Function for both Races by Stage

3.3 Comparison and Evaluation of Survivorship by Stage and Race

The approximate maximum likelihood estimates (MLE) and the corresponding estimated parametric survival and hazard functions for White and African American men classified by stage of the prostate cancer are shown in Table 3.3.1.

Table 3.3.1: Maximum Likelihood Estimates, Survival Functions, and Hazard Functions by the Stage of the Cancer and Race

Stage	Race	MLE	Estimated Survival Function	Estimated Hazard Function
I	Combine	$\hat{\kappa} = 3.6485$ $\hat{\lambda} = 2.5578$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.6485, \frac{t}{2.5578}\right)}{\Gamma(3.6485)}$	$\hat{h}(t) = \frac{t^{3.6485-1}e^{-t}}{\Gamma(3.6485) - \Gamma_t(3.6485)}$
	White	$\hat{\kappa} = 3.6485$ $\hat{\lambda} = 2.554$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.6485, \frac{t}{2.554}\right)}{\Gamma(3.6485)}$	$\hat{h}(t) = \frac{t^{3.6485-1}e^{-t}}{\Gamma(3.6485) - \Gamma_t(3.6485)}$

	African American	$\hat{\kappa} = 3.4172$ $\hat{\lambda} = 2.5709$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.4172, \frac{t}{2.5709}\right)}{\Gamma(3.4172)}$	$\hat{h}(t) = \frac{t^{3.4172-1}e^{-t}}{\Gamma(3.4172) - \Gamma_t(3.4172)}$
II	Combine	$\hat{\kappa} = 4.3383$ $\hat{\lambda} = 2.3173$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.3383, \frac{t}{2.3173}\right)}{\Gamma(4.3383)}$	$\hat{h}(t) = \frac{t^{4.3383-1}e^{-t}}{\Gamma(4.3383) - \Gamma_t(4.3383)}$
	White	$\hat{\kappa} = 4.3984$ $\hat{\lambda} = 2.2963$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.3984, \frac{t}{2.2963}\right)}{\Gamma(4.3984)}$	$\hat{h}(t) = \frac{t^{4.3984-1}e^{-t}}{\Gamma(4.3984) - \Gamma_t(4.3984)}$
	African American	$\hat{\kappa} = 3.715$ $\hat{\lambda} = 2.5553$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.715, \frac{t}{2.5553}\right)}{\Gamma(3.715)}$	$\hat{h}(t) = \frac{t^{3.715-1}e^{-t}}{\Gamma(3.715) - \Gamma_t(3.715)}$
III	Combine	$\hat{\kappa} = 5.7695$ $\hat{\lambda} = 1.9191$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(5.7695, \frac{t}{1.9191}\right)}{\Gamma(5.7695)}$	$\hat{h}(t) = \frac{t^{5.7695-1}e^{-t}}{\Gamma(5.7695) - \Gamma_t(5.7695)}$
	White	$\hat{\kappa} = 5.8528$ $\hat{\lambda} = 1.8993$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(5.8528, \frac{t}{1.8993}\right)}{\Gamma(5.8528)}$	$\hat{h}(t) = \frac{t^{5.8528-1}e^{-t}}{\Gamma(5.8528) - \Gamma_t(5.8528)}$
	African American	$\hat{\kappa} = 4.8089$ $\hat{\lambda} = 2.175$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.8089, \frac{t}{2.175}\right)}{\Gamma(4.8089)}$	$\hat{h}(t) = \frac{t^{4.8089-1}e^{-t}}{\Gamma(4.8089) - \Gamma_t(4.8089)}$

Table 3.3.1 Continued

Stage	Race	MLE	Estimated Survival Function	Estimated Hazard Function
IV	Combine	$\hat{\kappa} = 2.5843$ $\hat{\lambda} = 2.9435$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(2.5843, \frac{t}{2.9435}\right)}{\Gamma(2.5843)}$	$\hat{h}(t) = \frac{t^{2.5843-1}e^{-t}}{\Gamma(2.5843) - \Gamma_t(2.5843)}$
	White	$\hat{\kappa} = 2.7199$ $\hat{\lambda} = 2.8401$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(2.7199, \frac{t}{2.8401}\right)}{\Gamma(2.7199)}$	$\hat{h}(t) = \frac{t^{2.7199-1}e^{-t}}{\Gamma(2.7199) - \Gamma_t(2.7199)}$
	African American	$\hat{\kappa} = 1.7642$ $\hat{\lambda} = 3.7743$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(1.7642, \frac{t}{3.7743}\right)}{\Gamma(1.7642)}$	$\hat{h}(t) = \frac{t^{1.7642-1}e^{-t}}{\Gamma(1.7642) - \Gamma_t(1.7642)}$

Figure 3.3.1 represents the survival functions for the four stages of prostate cancer for Whites and African Americans. The graphs reveal differences between Whites and African Americans in each stage of prostate cancer compared with both races.

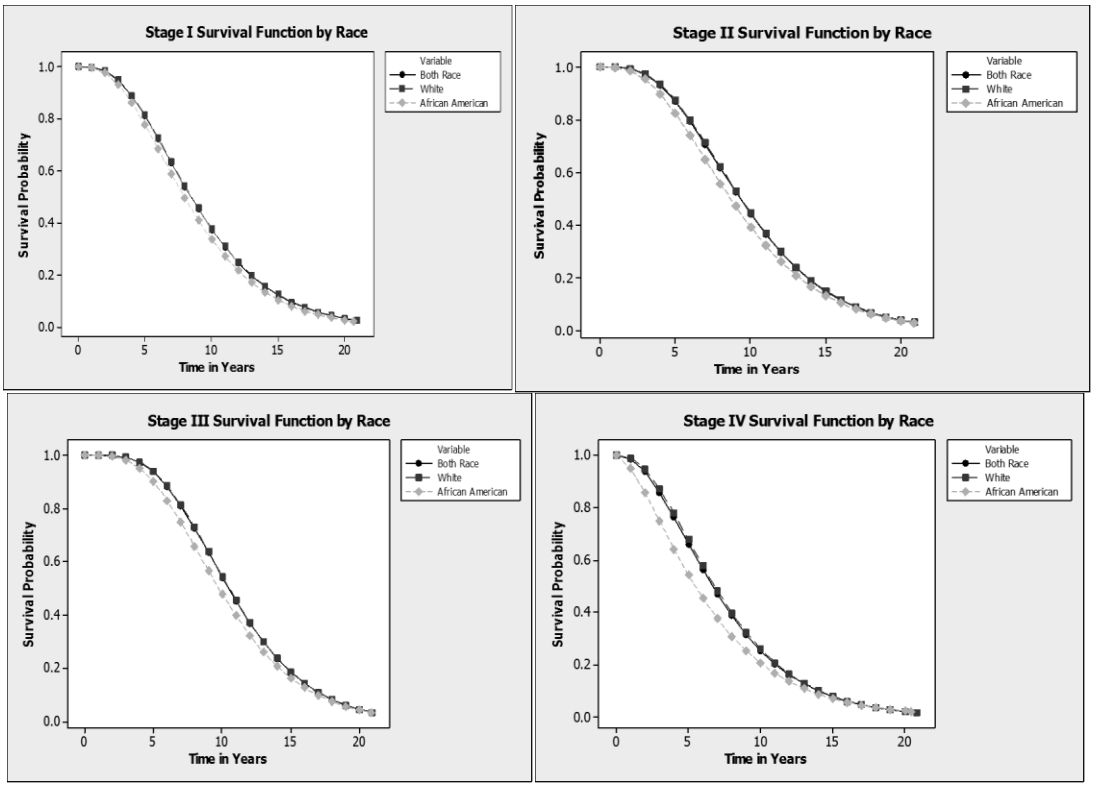

Figure 3.3.1: Survival Function by the Stage of the Prostate Cancer and Race

The estimated mean probabilities of the residuals for Whites in stage I, II, III, and IV compared with both race are approximately -0.23%, -0.22%, -0.21%, and -0.62%, respectively. On the other hand, the estimated mean probabilities of the residuals for African Americans in stage I, II, III, and IV compared with both races are approximately 2.20%, 2.59%, 2.86%, and 4.69%, respectively. This reveals that making a decision using combined races in each stage is underestimated for White men while it is overestimated for African American men in each stage. Thus, it is essential to identify the stage of the prostate cancer and race of the patient in order to make an efficient decision.

3.4 Comparison of Survivorship by Stage, Treatment, and Race

Stage I: The approximate maximum likelihood estimates (MLE) and the corresponding estimated parametric survival functions for White and African American men undergoing similar treatment are shown in Table 3.4.1.

Figure 3.4.1 represents the survival functions for Whites and African Americans undergoing similar treatment in stage I. The treatments considered under this subject matter are: Radiation therapy, Surgery, combination of Radiation and Surgery, and No Treatment.

The graphs reveal differences in survivorship between Whites and African Americans undergoing similar treatment. Under radiation therapy, surgery, or combination of radiation and surgery, White men appear to have a higher chance of survival compared with African American men. However, under no treatment, the chance of survival is higher in African American men than White men. The estimated average differences in survivorship between Whites and African Americans undergoing radiation

therapy, surgery, combination of radiation and surgery, and no treatment are 2.86%, 1.92%, 2.60%, and 1.38%, respectively.

Table 3.4.1: Maximum Likelihood Estimates and Estimated Survival Functions by Treatment and Race in Stage I

Treatment	Race	MLE	Estimated Survival Function
No Treatment	White	$\hat{\kappa} = 3.1912$ $\hat{\lambda} = 2.5031$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.1912, \frac{t}{2.5031}\right)}{\Gamma(3.1912)}$
	African American	$\hat{\kappa} = 2.2298$ $\hat{\lambda} = 3.216$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(2.2298, \frac{t}{3.216}\right)}{\Gamma(2.2298)}$
Radiation Therapy	White	$\hat{\kappa} = 4.9513$ $\hat{\lambda} = 1.872$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.9513, \frac{t}{1.872}\right)}{\Gamma(4.9513)}$
	African American	$\hat{\kappa} = 4.6351$ $\hat{\lambda} = 1.8825$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.6351, \frac{t}{1.8825}\right)}{\Gamma(4.6351)}$
Surgery	White	$\hat{\kappa} = 3.408$ $\hat{\lambda} = 2.8095$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.408, \frac{t}{2.8095}\right)}{\Gamma(3.408)}$
	African American	$\hat{\kappa} = 3.3341$ $\hat{\lambda} = 2.7503$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.3341, \frac{t}{2.7503}\right)}{\Gamma(3.3341)}$
Surgery & Radiation	White	$\hat{\kappa} = 4.0767$ $\hat{\lambda} = 2.4332$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.0767, \frac{t}{2.4332}\right)}{\Gamma(4.0767)}$
	African American	$\hat{\kappa} = 4.3071$ $\hat{\lambda} = 2.1639$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.3071, \frac{t}{2.1639}\right)}{\Gamma(4.3071)}$

Under all four different treatments, White men on average appear to have a longer survival time compared with African American men. The approximate difference in survival time under radiation therapy is 39 months, surgery is 8 months, combination of radiation and surgery is 7 months, and no treatment is less than 1 month.

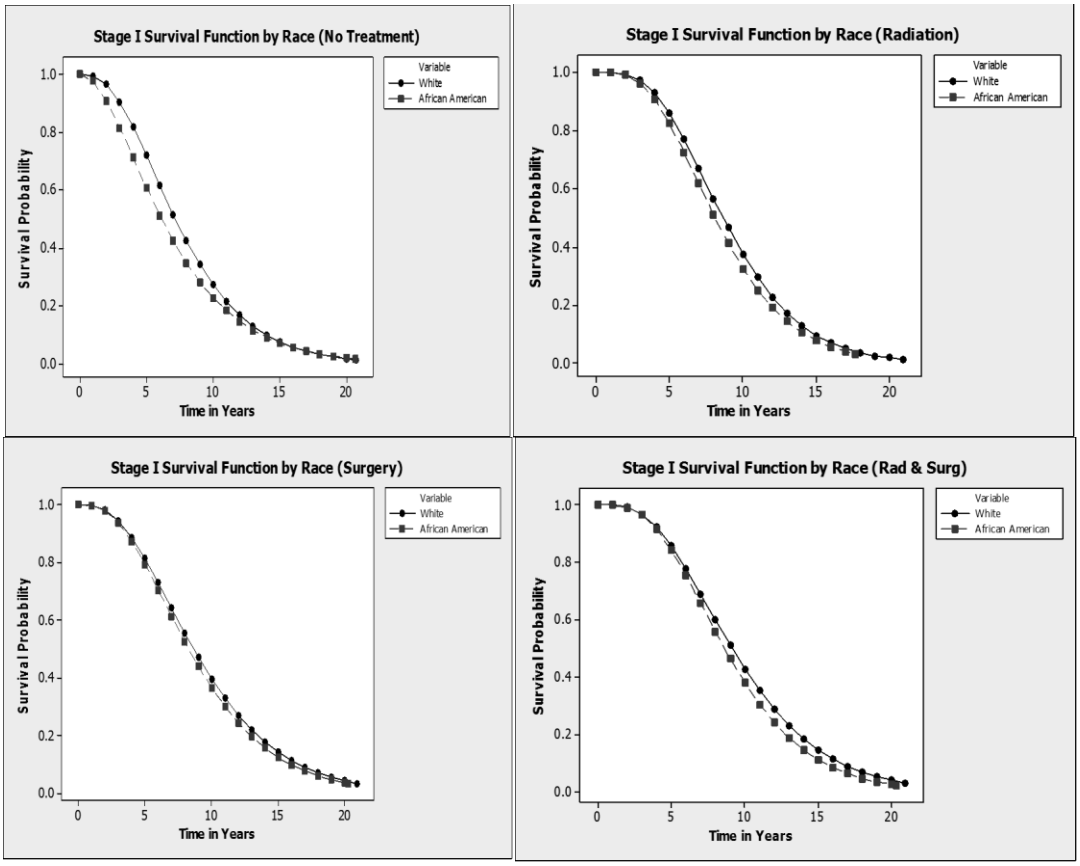

Figure 3.4.1: Survival Function by Treatment and Race in Stage I

Stage II: The approximate maximum likelihood estimates (MLE) and the corresponding estimated parametric survival functions for White and African American men undergoing similar treatment are shown in Table 3.4.2.

A graphical display of the survival functions for Whites and African Americans undergoing similar treatment in this stage of prostate cancer is given by Figure 3.4.2.

The figure reveals differences in survivorship between Whites and African Americans. White men undergoing surgery, combination of radiation and surgery, or no treatment appear to have a higher chance of survivorship compared with African American men. However, African American men receiving radiation therapy tend to have a higher chance of survivorship than White men. The estimated average differences in survivorship between Whites and African Americans undergoing radiation therapy, surgery, combination of radiation and surgery, and no treatment are 3.25%, 2.77%, 6.23%, and 10.35%, respectively.

Table 3.4.2: Maximum Likelihood Estimates and Estimated Survival Functions by Treatment and Race in Stage II

Treatment	Race	MLE	Estimated Survival Function
No Treatment	White	$\hat{\kappa} = 3.0899$ $\hat{\lambda} = 2.3659$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.0899, \frac{t}{2.3659}\right)}{\Gamma(3.0899)}$

	African American	$\hat{\kappa} = 1.6443$ $\hat{\lambda} = 3.4363$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(1.6443, \frac{t}{3.4363}\right)}{\Gamma(1.6443)}$
Radiation Therapy	White	$\hat{\kappa} = 4.1012$ $\hat{\lambda} = 2.2371$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.1012, \frac{t}{2.2371}\right)}{\Gamma(4.1012)}$
	African American	$\hat{\kappa} = 4.6364$ $\hat{\lambda} = 2.1293$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.6364, \frac{t}{2.1293}\right)}{\Gamma(4.6364)}$
Surgery	White	$\hat{\kappa} = 4.6908$ $\hat{\lambda} = 2.2259$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.6908, \frac{t}{2.2259}\right)}{\Gamma(4.6908)}$
	African American	$\hat{\kappa} = 4.1517$ $\hat{\lambda} = 2.3717$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.1517, \frac{t}{2.3717}\right)}{\Gamma(4.1517)}$
Surgery & Radiation	White	$\hat{\kappa} = 4.5783$ $\hat{\lambda} = 2.3841$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(4.5783, \frac{t}{2.3841}\right)}{\Gamma(4.5783)}$
	African American	$\hat{\kappa} = 3.2657$ $\hat{\lambda} = 2.9737$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.2657, \frac{t}{2.9737}\right)}{\Gamma(3.2657)}$

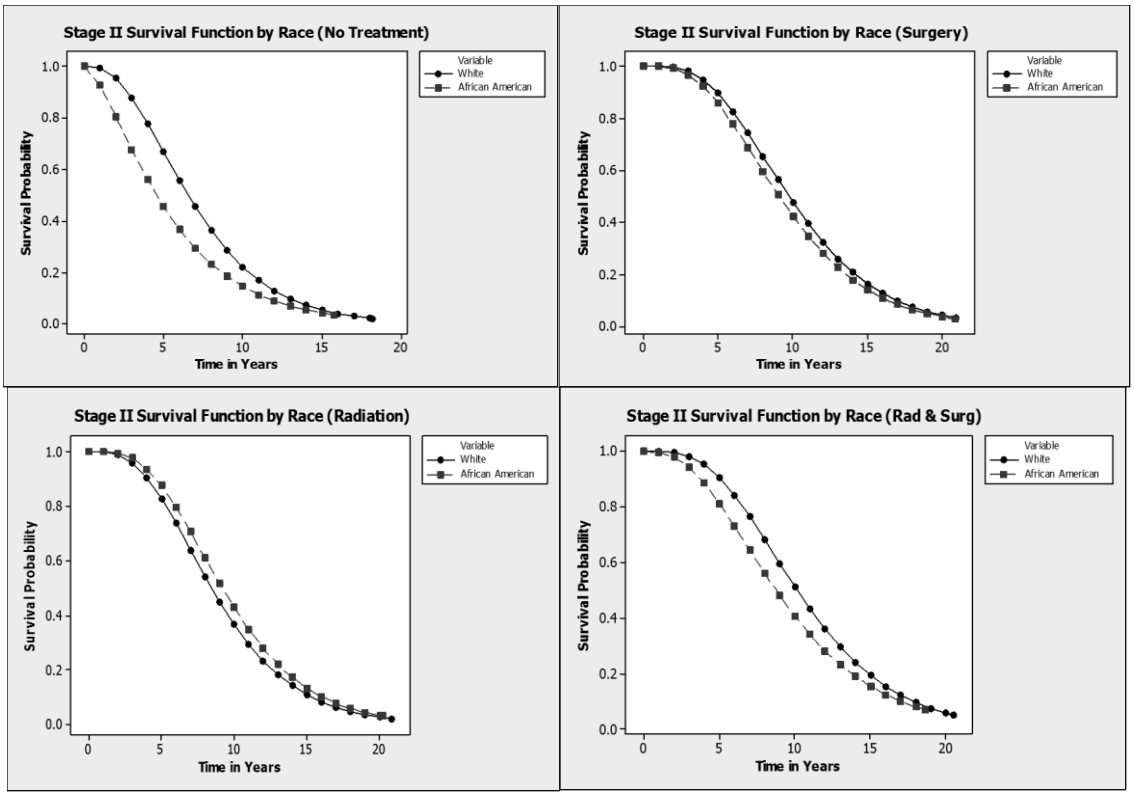

Figure 3.4.2: Survival Function by Treatment and Race in Stage II

White men on average appear to have a longer survival time compared with African American men under the four different treatments. The approximate difference in survival time under radiation therapy is 7 months, surgery is 1 month, combination of radiation and surgery is 23 months, and no treatment is 29 months.

Stage III: The approximate maximum likelihood estimates (MLE) and the corresponding estimated parametric survival functions for White and African American men undergoing similar treatment are shown in Table 3.4.3.

A graphical display of the survival functions for Whites and African Americans undergoing similar treatment in this stage of prostate cancer is given by Figure 3.4.3.

Table 3.4.3: Maximum Likelihood Estimates and Estimated Survival Functions by Treatment and Race in Stage III

Treatment	Race	MLE	Estimated Survival Function
No Treatment	White	$\hat{\kappa} = 1.938$ $\hat{\lambda} = 3.4314$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(1.938, \frac{t}{3.4314}\right)}{\Gamma(1.938)}$
	African American	$\hat{\kappa} = 2.221$ $\hat{\lambda} = 3.3091$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(2.221, \frac{t}{3.3091}\right)}{\Gamma(2.221)}$
Radiation Therapy	White	$\hat{\kappa} = 3.2585$ $\hat{\lambda} = 2.7088$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.2585, \frac{t}{2.7088}\right)}{\Gamma(3.2585)}$
	African American	$\hat{\kappa} = 3.531$ $\hat{\lambda} = 2.6489$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.531, \frac{t}{2.6489}\right)}{\Gamma(3.531)}$
Surgery	White	$\hat{\kappa} = 6.481$ $\hat{\lambda} = 1.758$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(6.481, \frac{t}{1.758}\right)}{\Gamma(6.481)}$
	African American	$\hat{\kappa} = 5.3341$ $\hat{\lambda} = 1.9879$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(5.3341, \frac{t}{1.9879}\right)}{\Gamma(5.3341)}$
Surgery & Radiation	White	$\hat{\kappa} = 5.5347$ $\hat{\lambda} = 2.0027$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(5.5347, \frac{t}{5.5347}\right)}{\Gamma(2.0027)}$
	African American	$\hat{\kappa} = 6.3647$ $\hat{\lambda} = 1.9051$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(6.3647, \frac{t}{1.9051}\right)}{\Gamma(6.3647)}$

Figure 3.4.3 reveals differences in survivorship between Whites and African Americans. In this stage, African American men undergoing radiation therapy, combination of radiation and surgery, or no treatment appear to have a higher chance of survivorship

compared with White men. However, White men undergoing surgery tend to have a higher chance of survivorship than African American men.

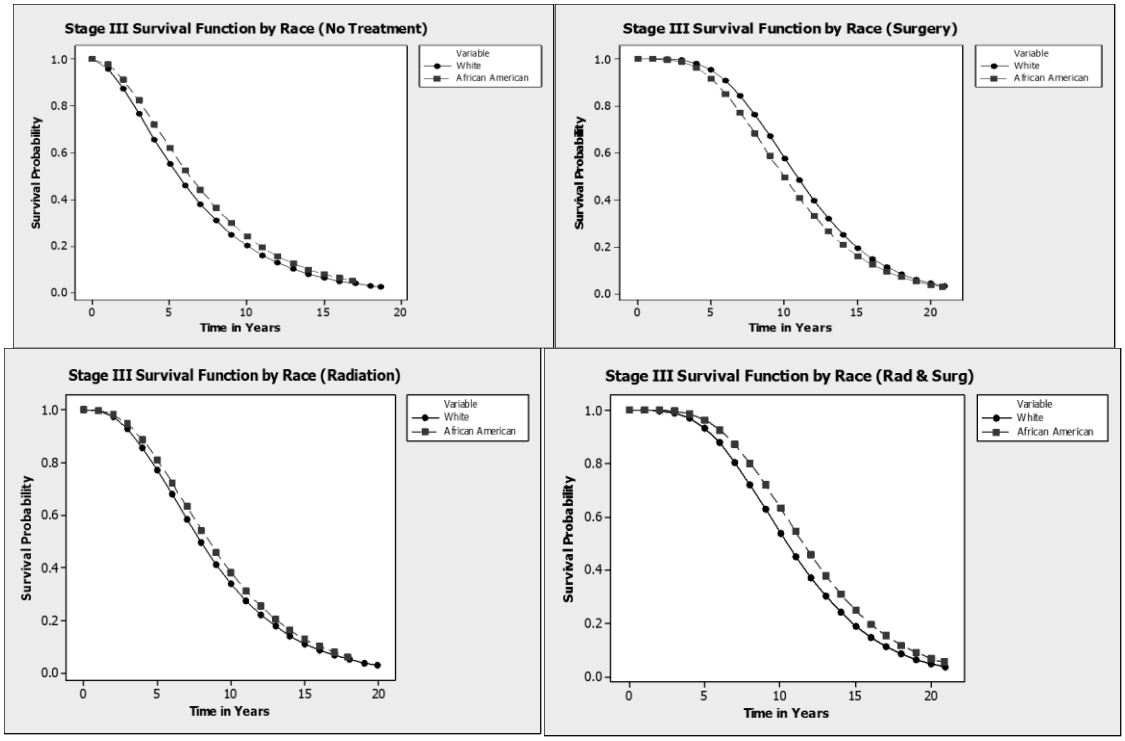

Figure 3.4.3: Survival Function by Treatment and Race in Stage III

The estimated average difference in survivorship for radiation therapy is 2.69%, surgery is 3.72%, combination of radiation and surgery is 4.69%, and no treatment is 3.85%. White men on average appear to have a longer survival time compared with African American men under the four different treatments. The approximate difference in survival time under radiation therapy is 24 months, surgery is 2 months, combination of radiation and surgery is 1 month, and no treatment is 22 months.

Stage IV: The approximate maximum likelihood estimates (MLE) and the corresponding estimated parametric survival functions for White and African American men undergoing similar treatment are shown in Table 3.4.4.

Table 3.4.4: Maximum Likelihood Estimates and Estimated Survival Functions by Treatment and Race in Stage IV

Treatment	Race	MLE	Estimated Survival Function
No Treatment	White	$\hat{\kappa} = 1.2609$ $\hat{\lambda} = 3.6218$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(1.2609, \frac{t}{3.6218}\right)}{\Gamma(1.2609)}$
	African American	$\hat{\kappa} = 1.0745$ $\hat{\lambda} = 4.378$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(1.0745, \frac{t}{4.378}\right)}{\Gamma(1.0745)}$
Radiation Therapy	White	$\hat{\kappa} = 1.8913$ $\hat{\lambda} = 3.5694$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(1.8913, \frac{t}{3.5694}\right)}{\Gamma(1.8913)}$

	African American	$\hat{\kappa} = 0.4346$ $\hat{\lambda} = 8.3444$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(0.4346, \frac{t}{8.3444}\right)}{\Gamma(0.4346)}$
Surgery	White	$\hat{\kappa} = 3.2735$ $\hat{\lambda} = 2.4762$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.2735, \frac{t}{2.4762}\right)}{\Gamma(3.2735)}$
	African American	$\hat{\kappa} = 2.3704$ $\hat{\lambda} = 3.1963$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(2.3704, \frac{t}{3.1963}\right)}{\Gamma(2.3704)}$
Surgery & Radiation	White	$\hat{\kappa} = 3.0529$ $\hat{\lambda} = 2.849$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.0529, \frac{t}{2.849}\right)}{\Gamma(3.0529)}$
	African American	$\hat{\kappa} = 3.6218$ $\hat{\lambda} = 2.1435$	$\hat{s}(t) = 1 - \frac{\Gamma_t\left(3.6218, \frac{t}{2.1435}\right)}{\Gamma(3.6218)}$

A graphical display of the survival functions for Whites and African Americans undergoing similar treatment in this stage of prostate cancer is given by Figure 3.4.4.

The graph reveals differences in survivorship between Whites and African Americans undergoing radiation therapy, and combination of radiation and surgery. For patients undergoing surgery, Whites appear to have a higher chance of survivorship than African Americans within the first 11 years. After this period, the survivorship appears to be approximately the same. Also, for patients with no treatment, the survivorship is approximately the same between both races.

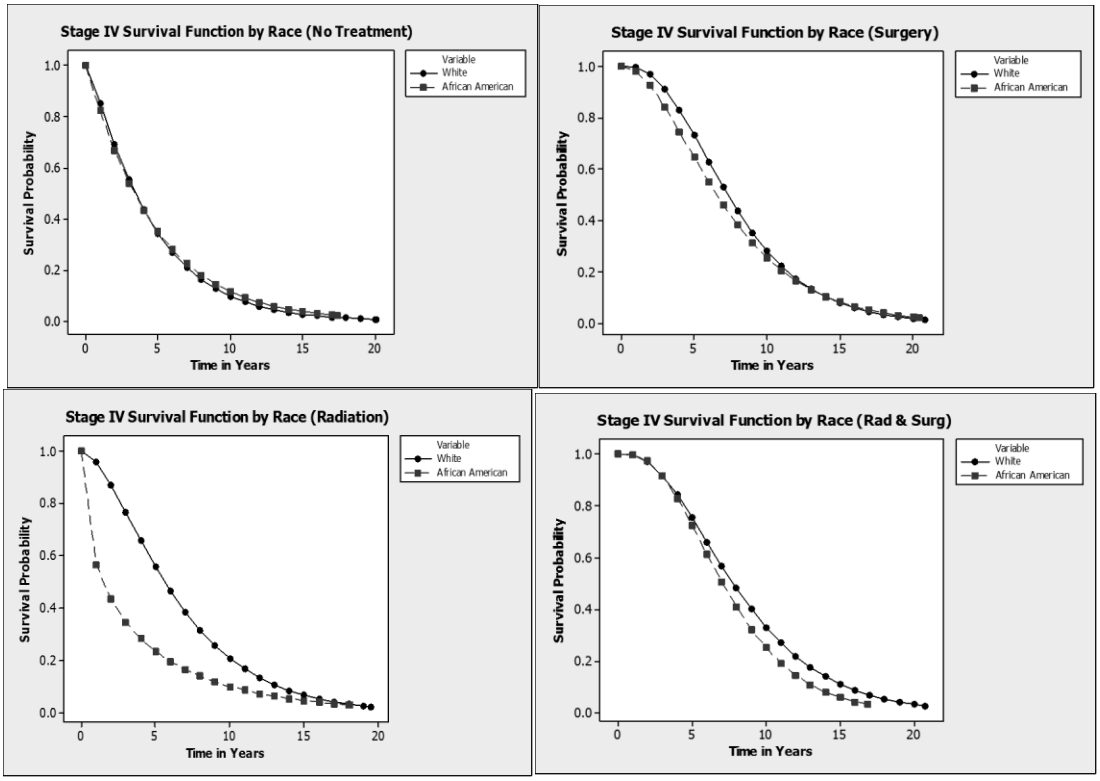

Figure 3.4.4: Survival Function by Treatment and Race in Stage IV

The estimated average difference in survivorship for radiation therapy is 16.35%, combination of radiation and surgery is 4.40%, and surgery in the first 11 years is 5.12%. White men on average appear to have a longer survival time compared with African American men under the four different treatments. The approximate difference in survival time under radiation therapy is 17 months, surgery is 5 months, combination of radiation and surgery is 47 months, and no treatment is 32 months.

4 CONCLUSIONS

In this present study, we can conclude that the average difference in survivorship of combined races is approximately 3.47% higher than African Americans. Also, the average differences in survivorship for all four stages compared with stage I, III, and IV are 2.37%, 5.79%, and 10.17%, respectively. Furthermore, the average differences in survival of African American men compared with both races in stage I, II, III, and IV are 2.20%, 2.59%, 2.86%, and 4.69%, respectively. These results reveal the existence of heterogeneity among race and the stage of the prostate cancer. Thus, in prostate cancer statistical analysis, it is essential to stratify by stage and race in order to make efficient decisions.

In stage I, White men have a better survival than African American men under radiation therapy, surgery, and combination of radiation and surgery. The corresponding average differences in survival are 2.86%, 1.92%, and 2.60%, respectively. However, under no treatment, African Americans have a better survival than Whites with a 1.38% average difference in survival. The survival time under radiation therapy is relatively smaller for African Americans than Whites.

In stage II, White men have a better survival than African American men under surgery, combination of radiation and surgery, and no treatment. The corresponding average differences in survival are 2.77%, 6.23%, and 10.35%, respectively. However, under radiation therapy, African Americans have a better survival than Whites with a 3.25% average difference in survival. The survival time under no treatment and combination of radiation and surgery are relatively smaller for African Americans than Whites.

In stage III, African American men have a better survival than White men under radiation therapy, combination of radiation and surgery, and no treatment. The average difference in survivorship under radiation therapy is 2.69%, combination of radiation and surgery is 4.69%, and no treatment is 3.85%. However, under surgery, Whites have a better survivorship than African Americans with a 3.72% average difference in survival. The survival time under radiation therapy and no treatment are relatively higher for Whites than African Americans.

In stage IV, White men have a better survival than African American men under radiation therapy and combination of radiation and surgery with an average difference in survivorship of 16.35% and 4.40%, respectively. In the first ten month period after surgery, White men have a better survival than African American men with 5.12% average difference in survival. However, there is no difference in survivorship between Whites and African Americans under no treatment. The survival time under radiation therapy, combination of radiation and surgery, and no treatment is relatively higher for Whites than African Americans.

In regards to these findings, physicians would be able to give an estimate of the survivorship between White and African American men under similar treatment at the four different stages of prostate cancer.

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